2017

AP CollegeBoard

AP Statistics

Sample Student Responses and Scoring Commentary

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AP® STATISTICS
2017 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion and (2) use a confidence interval for a proportion to find a confidence interval for a dollar amount that can be calculated using that proportion.

Solution

Part (a):

Step 1: Identify the appropriate confidence interval by name or formula and check appropriate conditions.

The appropriate procedure is a one-sample z-interval for a population proportion p. In this case, the population is all customers of the restaurant who ask for a water cup, and p is the proportion of that population who will fill the cup with a soft drink. The appropriate formula is $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

Conditions:

1. Random sample
2. Large sample (number of successes $n\hat{p} \geq 10$ and number of failures $n(1-\hat{p}) \geq 10$)

For condition 1 the stem of the problem states that a random sample of customers who asked for a water cup was used.

For condition 2 the number of successes (filled cup with soft drink) is 23 and the number of failures is 57, both of which are greater than 10.

Step 2: Correct mechanics

The sample proportion is $\hat{p} = \frac{23}{80} = 0.2875$. The confidence interval is

$$0.2875 \pm 1.96\sqrt{\frac{0.2875(1-0.2875)}{80}}$$
$$= 0.2875 \pm 1.96(0.0506) \quad \text{or } 0.1883 \text{ to } 0.3867.$$
$$= 0.2875 \pm 0.0992$$

Step 3: Interpretation

We can be 95 percent confident that in the population of all customers of the restaurant who ask for a water cup, the proportion who will fill it with a soft drink is between 0.1883 and 0.3867.

Part (b):

Using the confidence interval in part (a), a 95 percent interval estimate for the number of customers in June who asked for a water cup but then filled it with a soft drink is $3{,}000 \times 0.1883$ to $3{,}000 \times 0.3857$, or 565 to 1,160. At a cost of \$0.25 per customer, a 95 percent interval estimate for the cost to the restaurant in June is \$141.25 to \$290.00.

Question 2 (continued)

Scoring

This question is scored in four sections. Section 1 consists of step 1 in part (a), section 2 consists of step 2 in part (a), section 3 consists of step 3 in part (a), and section 4 consists of part (b). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the one-sample *z*-interval for a proportion is identified (either by name or formula) *AND* both conditions (random sampling and large sample) are adequately addressed.

Partially correct (P) if the response identifies the correct procedure *BUT* adequately addresses only one of the two required conditions;
OR
if the response does not identify the correct procedure *BUT* adequately addresses both required conditions.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Stating the large sample condition without verifying it is not sufficient. The response must use specific numerical values to adequately address the condition.
- If the response includes additional inappropriate conditions, such as $n \geq 30$ or requiring a normal population, then the response earns at most P for section 1.
- Stating and checking a condition about the size of the sample relative to the size of the population is appropriate but not required.
- Any statement of hypotheses, description of the population, or definition of the parameter should be considered extraneous. However, if such statements are included and incorrect, they are considered as poor communication in terms of holistic scoring.

Section 2 is scored as follows:

Essentially correct (E) if the response gives the correct 95 percent confidence interval. Supporting work is not required, but if included, it must be correct.

Partially correct (P) if the response gives a correct confidence interval with incorrect (but appropriate) supporting work shown;
OR
if the response gives an incorrect but reasonable confidence interval with appropriate supporting work shown — for instance, if a value other than 1.96 is used for the critical value or a value other than $\frac{23}{80}$ is used for $\hat{p}$.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Appropriate supporting working must have the form:
  proportion ± (critical value)(SE/SD of proportion).

Question 2 (continued)

- A confidence interval that has both endpoints outside the interval from 0 to 1 is considered unreasonable.

Section 3 is scored as follows:

Essentially correct (E) if the response provides an appropriate interpretation of the interval that includes the following three components:

1. Conveys inference about a population proportion
2. Demonstrates a clear understanding that the parameter is the proportion of the water cup population that fills the cup with a soft drink
3. Mentions 95 percent confidence and interprets it correctly using words such as "We can be 95 percent confident" or "With 95 percent confidence"

Partially correct (P) if the response provides an appropriate interpretation of the interval that includes the first component *AND* only one of the other two components;
OR
if the response provides a correct interpretation of the confidence *level* in context without interpreting the specific interval.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Clear indication of an inference to the random sample of 80 customers rather than to the population does not satisfy the first component and is scored I.
- Stating values that are unrealistic as proportions or percentages (including blanks) in the interpretation lowers the score one level (from E to P, or from P to I).
- When both the interpretation and the level of the interval are given, only the interpretation is scored. If the interpretation of the confidence level is incorrect, it is considered as poor communication in terms of holistic scoring.
- Any interpretation that implies the interval has a 95 percent chance (or possibility or probability) of capturing the population proportion is scored I.

Section 4 is scored as follows:

Essentially correct (E) if the response gives a correct interval estimate for the cost to the restaurant *AND* shows enough work to indicate how the interval was found.

Partially correct (P) if the response gives the correct interval estimate for the *number* of customers (565 to 1,160) who would fill the water cup with soda, including showing work, but does not multiply by \$0.25 to find the interval for the cost;
OR
if the response gives the correct interval estimate for the *expected cost* to the restaurant for an individual who asked for a water cup (\$0.05 to \$0.10), including showing work, but does not multiply by 3,000 to find the interval for the total cost;
OR
if the response makes a reasonable attempt to find the endpoints of the interval as $(0.1883)(3{,}000)(0.25)$ and $(0.3867)(3{,}000)(0.25)$, but makes an error such as using the sample size of 80 instead of the population size of 3,000;

Question 2 (continued)

OR
if the response gives the correct interval without showing how it was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The response in section 4 earns an E if it follows correctly from an incorrect interval found in part (a), even if the interval is unreasonable.
- Units ($) are not required to earn an E.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and communication.

2A

2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.

(a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

Step 1: let p = the proportion of all customers who fill the cup with soda rather than water when asking for water

Step 2: 1 proportion z interval

1. The manager selected a random sample ✓
2. independent: $10n_1 < pop$ $10(80) \leq 800$ $800 < pop$ ✓
3. Normal: $np > 10$ $n(1-p) > 10$

$(.2875)(80) = 23$ ✓ $80(1-.2875)(80) = 57$ ✓

Step 3: (.18832, .38668)

Step 4: We are 95% confident that the interval from .18832 to .38668 will capture the true population proportion of all customers who fill the cup with soda rather than water when asking for water.

(b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant $0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

$3,000\ (.18832) = 564.96$ $564.96\ (.25) = 141.24$

$3,000\ (.38668) = 1160.04$ $1160.04(.25) = 290.01$

In the month of June, the cost to the restaurant will be between $141.24 and $290.01 using the ~~[illegible]~~ 95% confidence interval from part (a).

2B

2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.

(a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

$23/80 = .2875$

$.2875 \pm 1.96 \times .0506$

$.2875 \pm .099176$

$\text{InvNorm}(.975, 0, 1) = 1.96$

$\sqrt{\frac{.2875 \times .7125}{80}} = .0506$

$(.188324, .386676)$

We are 95% confident that the true proportion of customers who ask for a water but fill their cup with pop is between .188324 & .386676.

(b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant $0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

$3000 \times .188324 \approx 565$ $\quad 565 \times .25 = 141.25$

$3,000 \times .386676 \approx 1160$ $\quad 1160 \times .25 = 290$

($141.25, $290)

2C

2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.

(a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

One proportion Z test

$\hat{p} = \frac{23}{80} = .2875$ $\sigma = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .0506$

$Z = 1.96$ for 95% confidence

$\hat{p} \pm Z \cdot \sigma$ $\hat{p} \pm 1.96(.0506)$

(.18832, .38668)

If we were to measure all possible samples of this size, we are confident the sample proportion would fall between 0.18832 to 0.38668, 95% of the time

(b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant $0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

(.18832 · 3000, 0.38668 · 3000)

# of customers who get soda with water cup: (564.96, 1160.04)

$ amount ⇒ (141.24, 290.01)

(# of customers who use water cup for soda × 0.25)

We are 95% confident the $ spent on giving water cups for soda drinkers is between $141.24 and $290.01.

AP® STATISTICS
2017 SCORING COMMENTARY

Question 2

Overview

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion and (2) use a confidence interval for a proportion to find a confidence interval for a dollar amount that can be calculated using that proportion.

Sample: 2A
Score: 4

The response in part (a) correctly identifies the procedure as a "1 proportion z interval" and addresses both the random sampling condition and the large sample condition ($n\hat{p} \geq 10$ and $n\hat{q} \geq 10$) adequately, so section 1 was scored as essentially correct. Note that the response also states and checks a condition about the sample size relative to the population size, which is appropriate but not required. The correct 95 percent confidence interval is given, so section 2 was scored as essentially correct. The response provides a correct interpretation of the confidence interval that includes all three required components — inference about a population proportion, correct parameter definition, and 95 percent confidence. As a result section 3 was scored as essentially correct. In part (b) the response uses the interval from part (a) to calculate a correct interval estimate for the cost to the restaurant in the month of June for customers who ask for a water cup but fill the cup with a soft drink, with sufficient work shown. Consequently, section 4 was scored as essentially correct. Because four sections were scored as essentially correct, the response earned a score of 4.

Sample: 2B
Score: 3

The response correctly identifies the procedure using an appropriate formula in part (a). However, neither the random sampling condition nor the large sample condition is addressed. As a result section 1 was scored as incorrect. The correct 95 percent confidence interval is given along with appropriate supporting work, so section 2 was scored as essentially correct. An appropriate interpretation of the confidence interval that includes all three required components is provided. Consequently, section 3 was scored as essentially correct. In part (b) the response gives a correct interval estimate for the cost with work shown for multiplying both endpoints of the interval from part (a) by 3,000 and $0.25, so section 4 was scored as essentially correct. Because three sections were scored as essentially correct, and one section was scored as incorrect, the response earned a score of 3.

Sample: 2C
Score: 2

The response incorrectly identifies the appropriate procedure as a "one proportion z test" in part (a). Also, neither the random sampling condition nor the large sample condition is addressed. Consequently, section 1 was scored as incorrect. The correct 95 percent confidence interval is given along with appropriate supporting work, so section 2 was scored as essentially correct. An incorrect attempt is made to interpret the confidence level, and because no interpretation of the confidence interval is provided, section 3 was scored as incorrect. The response gives a correct interval estimate for the cost in part (b) with enough work shown, so section 4 was scored as essentially correct. Because two sections were scored as essentially correct, and two sections were scored as incorrect, the response earned a score of 2.